Universe restriction in the logic of language

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It is my pleasure to dedicate this study to my old friend and colleague Frans Zwarts, whose interest in the vagaries of negation in natural language is almost as old as mine.

*Abstract: Negation in language, far from being a simple representative of standard bivalent negation, has its own idiosyncrasies. The present study is an attempt at subsuming many or most of these under the rubric of universe (**Un**) restriction, meaning that natural language negation selects different complements under different conditions. **Un** restriction is manifest in three different ways: static, dynamic and developmental. The focus of this study is on the static form of **Un** restriction, leading to an investigation of the relation between unrestricted standard modern predicate logic on the one hand and the classic Square of Opposition on the other. It is argued further that the addition of a 'strict' existential quantifier 'some but not all' leads to increased insight into the matter.*

1. Introduction

Although there can be no doubt about the universal mathematical validity of standard modern predicate logic (SMPL), the status of uniqueness and untouchability it has acquired over the past century is unwarranted and has proved an obstacle to deeper insights into the richness of logical space and in particular its relation with language and cognition. Human cognition, with language in its wake, has made a far more ingenious use of the possibilities afforded by logical space than present-day logicians, semanticists and psychologists have given it credit for. The present study explores the way in which cognition has found its way in logical space, with an emphasis on the one outstanding feature of the resulting logic of natural language (LNL), its tendency to reduce SMPL's infinite universe (**Un**$^{\omega}$)of all metaphysically possible situations to functionally manageable proportions. Whereas SMPL operates in terms of one unchangeable and infinite universe of discourse, LNL operates within universes that are, for obvious functional reasons, restricted in a variety of ways. Although this principle was already emphasized at great length in the chapters 2 and 4 of De Morgan (1847), it is only now that the various aspects of this basic fact are beginning to come to light, and as this is happening, it is becoming clear that the well-known discrepancies between SMPL and natural logical intuitions are to be accounted for in terms of **Un** restriction, rather than in terms of pragmatic (Gricean) principles, as is commonly held nowadays.

The phenomenon of **Un** restriction manifests itself, as far as can be seen today, in three distinct but interrelated ways: *statically*, *dynamically* and *developmentally*. The common thread to all three forms of **Un** restriction is the way negation selects a complement in any given circumstance: the complement selected by the negation will be more restricted as **Un** gets more constrained. In the present study it is only the *static* aspect that is considered and analysed. Yet, for the ideas presented here to be seen in their proper context, a few words must be said about dynamic and developmental **Un** restriction.

*Dynamic **Un** restriction* is present mainly in the phenomenon of *presupposition*, which has been studied systematically only since the middle of last century. The main feature of presuppositions as semantic properties of (type-level) sentences is that they are discourse restrictors: a sentence S presupposing P is usable only in discourse (sub)domains **D** where P has been explicitly added or can be supplemented on grounds of situational or world knowledge. If **D** does not meet this condition, S 'bounces'. The behaviour of negation shows that this **D**-restriction is at the same time a **Un** restriction: the restricted **D** acts as the restricted **Un** in the running discourse, which makes the negation presupposition-preserving (Seuren 1972). In fact, it was this property of negation in natural language that gave rise to presupposition theory. Another indicator is conjunction: the difference between *He went to Spain and got rich* on the one hand and *He got rich and went to Spain* on the other is precisely that the second sentence in each case is interpreted in a **Un** to which the first sentence has been added. Iconicity, usually adduced as a ground of explanation, is merely coincidental: the real reason for the difference lies in the progressive restriction of **Un** as the discourse goes on. This form of **Un** restriction is called *dynamic* because it makes **Un** follow the *ad hoc* movements of any running discourse. I will not go into the complex issues related to presupposition theory in this brief study. For detailed discussion, analyses and arguments see, for example, Seuren (1985, 2010).

*Developmental **Un** restriction* is a phenomenon that has only recently become the object of research. The context of this research is formed by the question of why some negative lexicalizations are systematically absent from the lexicons of natural languages. It started with an observation by Thomas Aquinas (1225–1274) in his *Expositio Peryermeneias* lib. 1.1.10, n. 13 (Horn 1972, Ch. 4) that while logical *some* has a negative counterpart in *no*, *all* lacks such a lexical counterpart. This fact caught fire recently, resulting in a spate of publications (e.g. Blanché 1953:95–6; Horn 1989:253; Löbner 1990:95; Levinson 2000:69–71), where the observation was widened to the truth functions and the epistemic modalities. *Or* has a corresponding negative *neither…nor* but *and* lacks a negative counterpart (**nand* only occurs in the technical vocabulary of computer science); epistemic *possible* has a negative opposite in epistemic *impossible* but epistemic *necessary* lacks an epistemic negative counterpart (*unnecessary* is not epistemic). In all such cases, it is the contradictory of the element in the **A**-position of the Square structure (consisting of the proposition types **<A**, **I**, **¬I**, **¬A>**) these items fit into that withstands lexicalization. This fact has been dubbed the *unlexicalisability of the **O** corner*. Also, for example, the quadruple **<**order, permit, forbid, *non-order**>**, which logically corresponds to the traditional Square: *order* entails *permit*, *forbid* is the **Un**-restricted contradictory of *permit* and **non-order* of *order*, *order* and *forbid* are contraries and, finally, *permit* and **non-order* are subcontraries. Typically, there is no lexical item for the **O**-corner item **non-order* in the logically expected sense of ‘either forbid or permit’. And analogously for **<**cause, allow, prevent, *non-cause**>** in the logic of causality.

It was found recently (Jaspers 2012; Seuren & Jaspers, forthcoming; Seuren, forthcoming) that the regularity pervades the lexicon as a whole: in any lexical assembly whose members can be arranged in the form of the classic Square, the **O** position remains unlexicalised. To mention just one example out of a myriad, in Ancient Greek, which has a word *Athenian*, entailing *Greek*, and a word *barbarian* for ‘non-Greek’, no word exists for the **O**-corner item ‘non-Athenian’ covering both non-Athenian Greeks and barbarians. This non-lexicalisability of items in the **O** position appears to be universal for ordinary, non-technical language.

This regularity is reinforced when the Square is extended with two additional vertices, one for the strict existential quantifier ‘some but not all’ (the **Y**-type) and one for its contradictory (the **¬Y**-type, often called **U**). As shown below, this extends the Square to the *logical Hexagon*, as proposed in Jacoby (1950, 1960), Sesmat (1951), Blanché (1953, 1966). In the Hexagon it is not only the **¬A** (=**O**) position but also the **¬Y** (=**U**) position that is unlexicalisable. Moreover, the (unlexicalised) negation of *all*, as in *Not all children laughed*, takes one to ‘some but not all children laughed’, rather than to the expected ‘either no or some but not all children laughed’.

In order to account for this class of observations, a cognitive mechanism is postulated (Seuren & Jaspers, forthcoming; Seuren, forthcoming) in virtue of which **Un** is stepwise and recursively restricted in the course of early cognitive development—as a result of increased cognitive activity—so that the (default) negation selects different complements according to the degree to which the logical system has been developed. Assume that the young child’s very first distinction, in the world of its experiences, will be between there being nothing and there being something. Subsequently, the child will discover that not all entities (‘somethings’) are equal, as some have this and others that property. Then, when combining properties, the child discovers that, given entities with property R, some or none of these also have property M, thus establishing the first or primary axis of a predicate-logical system, consisting of the opposition between the proposition types **I** (Some R is M) and **¬I** (No R is M). But this mental operation takes place within, or on the basis of, the class of experiences that there are some entities with the property R. Then, after a number of experiences of ''Some R is M', the child will jump to either 'All R is M' or 'Not all R is M'. But this distinction is made within the class of experiences that (at least) some R is M. This throws some light on the fact that a sentence such as *Not all children laughed* is naturally interpreted under the assumption that at least some children laughed—that is, as 'some children laughed and some did not', the **Y**-type.

The matter is complicated by the fact that the entire hypothesis of static, dynamic and developmental **Un** restriction is subject to the *caveat* that culture-induced intellectual development is capable of undoing **Un** restrictions, arriving, in the end, at a totally unrestricted, contingency-free logical system such as SMPL, which took over twenty centuries of culture to come about and which is rightly regarded as an intellectual triumph. The Square (in the form in which it was handed down by tradition) is an earlier, incomplete, product of such culture-induced undoing of **Un** restriction, as it dispensed with the restrictions imposed by developmental and dynamic **Un** restriction, but failed to remove the static restriction that the R-class must be nonnull.

As has been said, however, it is not my purpose here to elaborate the details of dynamic and developmental **Un** restriction, but they form the background to the more precise analysis of static **Un** restriction which is the topic of the present study.

2. The null R-class in standard modern predicate logic

From a strictly logical point of view, as is well known, the Square suffers from the defect that it becomes inconsistent when applied to cases where the class denoted by the subject term (the R-class) is empty: the Square is valid only for cases where the R-class is nonnull. This defect, known as *Undue Existential Import* (UEI), is remedied by SMPL, which, as befits an ideal logical system, is independent of contingencies and valid in an infinite $\mathbf{Un}^{\omega}$, encompassing all possible situations, including those where the R-class is null. In this section, I argue that the Square's singling out of the condition that the R-class be nonnull (a) is not an arbitrary defect but cuts into SMPL at a mathematically well-defined joint in the system, (b) is motivated by the fact that it maximally increases the logical power of the system, and (c) makes perfect sense in a reconstruction of the logic of natural cognition, as the null set does not seem to have cognitive reality (Seuren 2010, Ch. 3). These points have never been observed, let alone been properly appreciated.

Both the Square and SMPL are predicated on two types of quantified propositions, type **A** (*All R is M*) and type **I** (*At least some R is M*), where the variables *R* (*restrictor predicate*) and *M* (*matrix predicate*) range, in principle, over all first order predicates. In addition, both systems incorporate an *external* and an *internal negation*. The former negates entire propositional types and is symbolized here as **¬** prefixed to the type name. The latter negates the propositional function in M-position and is symbolized as ***** suffixed to the type name. This gives eight type names, as shown in Table 1.

Table 1

type name	linguistic expression	formal language
A	All R is M	$\forall x(Rx,Mx)$
I	At least some R is M	$\exists x(Rx,Mx)$
¬A	Not all R is M	$\neg[\forall x(Rx,Mx)]$
¬I	No R is M	$\neg[\exists x(Rx,Mx)]$
A*	All R is not M	$\forall x(Rx,\neg Mx)$
I*	At least some R is not M	$\exists x(Rx,\neg Mx)$
¬A*	Not all R is not M	$\neg[\forall x(Rx,\neg Mx)]$
¬I*	No R is not M	$\neg[\exists x(Rx,\neg Mx)]$

The type names are merely a convenient shorthand; they are not meant to be a formal logical language. The logical language used throughout this study is, in principle, that of the *theory of generalized quantifiers*, where the quantifiers are taken to be binary higher-order predicates, as exemplified in the column "formal language" of Table 1. Any claim that SMPL is superior to the Square in that the former is able to express multiple internal quantification, as in *Some boys admire all popsingers*, while the latter lacks that expressive power, is thus false, as it is based on a confusion of the logical system itself on the one hand and the formal language used on the other. In principle, the same formal language can be used for both SMPL and the Square, or indeed any other variety of predicate logic.

In the language used, the quantifiers $\forall$ and $\exists$ are higher order predicates over pairs of first-order sets. *Rx* and *Mx* are first-order set-denoting predicates (propositional functions). The quantifying predicates $\forall$ and $\exists$ are semantically defined as in (1) (the variable *x* binds the *x* in *Rx* and *Mx*; $[\![P]\!]$ stands for the appropriate order extension of any predicate *P* in the universe of entities **Ent**):

(1) a. $\llbracket\forall\rrbracket = \{<\llbracket R\rrbracket,\llbracket M\rrbracket> \mid \llbracket R\rrbracket \subseteq \llbracket M\rrbracket\}$

b. $[\exists\rrbracket = \{<\llbracket R\rrbracket,\llbracket M\rrbracket> \mid \llbracket R\rrbracket \cap \llbracket M\rrbracket \neq \emptyset\}$

That is, *All R is M* is read as 'the class of Rs is included in the class of Ms', while *Some R is M* is read as 'there is a nonnull intersection between the class of Rs and the class of Ms'.

Given (1), **A** ≡ **¬I*** and **I** ≡ **¬A***. These equivalences are known as the *Conversions*, and the quantifiers $\forall$ and $\exists$ are commonly called *duals*. The proof is easy:

for **A** ≡ **¬I***: **A** is true iff $\llbracket R\rrbracket \subseteq \llbracket M\rrbracket$; **¬I*** is true iff $\llbracket R\rrbracket \cap \llbracket \overline{M}\rrbracket = \emptyset$; $\llbracket R\rrbracket \subseteq \llbracket M\rrbracket$ and $\llbracket R\rrbracket \cap \llbracket \overline{M}\rrbracket = \emptyset$ express the same set-theoretic situation; hence, **A** ≡ **¬I***.

for **I** ≡ **¬A***: **I** is true iff $\llbracket R\rrbracket \cap \llbracket M\rrbracket \neq \emptyset$; **¬A*** is true iff $\llbracket R\rrbracket \not\subseteq \llbracket \overline{M}\rrbracket$; $\llbracket R\rrbracket \cap \llbracket M\rrbracket \neq \emptyset$ and $\llbracket R\rrbracket \not\subseteq \llbracket \overline{M}\rrbracket$ express the same set-theoretic situation; hence **I** ≡ **¬A***.

The Conversions define SMPL. Beyond the Conversions, **A** and **I** are logically independent in SMPL in the sense that they can be true or false independently of each other (provided neither **A** nor **I** is either necessarily true or necessarily false). There are thus, for contingent **A** and **I**, four possible valuations (situations) in the universe **Un** of admissible valuations, as shown in the first two rows of Table 2 ('T' stands for True; 'F' for False). The six remaining rows follow from the first two in virtue of the truth-functional definition of standard negation, whether external or internal.

Table 2

Un:	1	2	3	4
A	T	F	F	T
I	T	T	F	F
¬A	F	T	T	F
¬I	F	F	T	T
A*	F	F	T	T
I*	F	T	T	F
¬A*	T	T	F	F
¬I*	T	F	F	T
Conditions for T:	$[\![R]\!] \subseteq [\![M]\!]$ $[\![R]\!] \cap [\![M]\!] \neq \emptyset$	$[\![R]\!] \not\subseteq [\![M]\!]$ $[\![R]\!] \cap [\![M]\!] \neq \emptyset$	$[\![R]\!] \not\subseteq [\![M]\!]$ $[\![R]\!] \cap [\![M]\!] = \emptyset$	$\boldsymbol{[\![R]\!] = \emptyset}$

Each of the four valuations in Table 2 specifies a truth value (T or F) for each of the eight proposition types in SMPL, plus the conditions to be fulfilled for truth in each valuation. These truth conditions are simply derived from the satisfaction conditions specified for the quantifying predicates $\forall$ and $\exists$ in (1) above. For valuation 4, this condition is the conjunction of $[\![R]\!] \subseteq [\![M]\!]$ (since **A** is true) and $[\![R]\!] \cap [\![M]\!] = \emptyset$ (since **I** is false), which amounts to the simple condition that $[\![R]\!] = \emptyset$. That this is so is easily shown: for any arbitrary $[\![R]\!]$ and $[\![M]\!]$, if $[\![R]\!] = \emptyset$, $[\![R]\!] \subseteq [\![M]\!]$ and $[\![R]\!] \cap [\![M]\!] = \emptyset$; conversely, if $[\![R]\!] \subseteq [\![M]\!]$ and $[\![R]\!] \cap [\![M]\!] = \emptyset$; then $[\![R]\!] = \emptyset$. Note that the conditions for truth in the valuations 1,2 and 3 all entail that $[\![R]\!] \neq \emptyset$.

This fact has not or hardly received any attention in the literature, yet it is remarkable in that situations where $[\![R]\!] = \emptyset$ form the crucial difference between SMPL and the Square: the two systems are identical but for the fact that SMPL (in conformity with standard set theory) treats **A**-type propositions as true when $[\![R]\!] = \emptyset$, while the Square ignores such situations. This shows that leaving cases where $[\![R]\!] = \emptyset$ out of consideration is not an arbitrary decision but cuts into SMPL at a natural joint (a fact that does not hold for Aristotelian-Abelardian logic; see below). Dropping cases where $[\![R]\!] = \emptyset$ from **Un** creates a wealth of new logical relations giving rise to the Square and giving the system a logical power unsurpassed by any other known logical system in terms of the eight proposition types defined in Table 1. Moreover, in the context of a reconstruction of the natural logical powers of the human race it is relevant to observe that leaving out cases where ⟦R⟧ = Ø is cognitively plausible in that what is known in mathematics as the null set seems to lack cognitive reality in unsophisticated humans, being the product of highly developed mathematical abstraction (Seuren 2010, Ch. 3).

Table 2 can be converted into a diagram as in Figure 1, consisting of four concentric spaces, each space being marked for the proposition types that are true in it. This way of representing logical systems is called *Valuation Space (VS) analysis* and the corresponding diagrams are called *VS-models*. The following section expands on these notions.

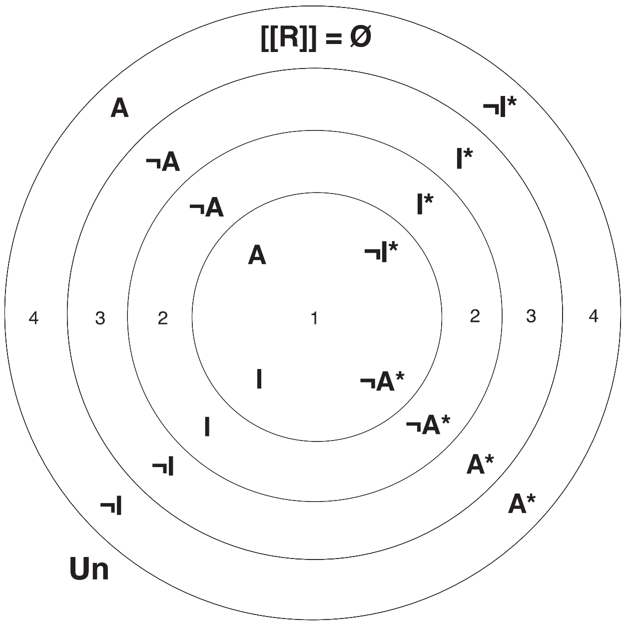

Figure 1 VS-model of SMPL

3. Valuation Space analysis

Before looking more closely into the logical properties of the Square, let us first consider the proof method for logical systems called *Valuation Space analysis*, already instantiated in Table 2 and Figure 1 above. Let /P/ denote the *Valuation Space* (VS) of any proposition (or proposition type) *P*, that is, the set of situations/valuations in the universe **Un** of possible situations in which *P* is true, given the propositions (or proposition types) considered, under standard bivalence (in the present study).[1] The basic way of defining a situation is by means of assigning truth values to the (types of) propositions in the language fragment **L** considered. Thus, if **L** contains exactly three logically independent propositions *A*, *B* and *C*, **Un** consists of eight (2^3) possible situations/valuations (see Table 3). (If **L** consists of 25 logically independent propositions, the number of valuations is 2^{25}, or 33,554,432.) Each of the situations/ valuations 1–8 is an assignment of truth values to *A*, *B* and *C*. The logical compositions follow automatically, given the definitions of the propositional operators. Thus, the VS /A/ of proposition *A* in **Un** as specified in Table 3 is {1,3,5,7}, /B/ = {1,2,5,6}, etc. (For more elaborate discussion see Seuren 2013, Ch. 8.)

Table 3

Un:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
A	T	F	T	F	T	F	T	F
B	T	T	F	F	T	T	F	F
C	T	T	T	T	F	F	F	F
¬A	F	T	F	T	F	T	F	T
A∧B	T	F	F	F	T	F	F	F
A∨B	T	T	T	F	T	T	T	F
A∨C	T	T	T	T	T	F	T	F

etc.

The diagrammatic representation of SMPL given in Figure 1 can thus be interpreted as a *VS-model* for SMPL, with the four spaces corresponding to the four valuations of Table 2.

When the members of a pair of propositions considered are not logically independent, so that there exists a logical relation between them, some situations are *inadmissible*. For example, if B entails C (B ⊢ C), all situations in which B is valued true and C is valued false are inadmissible, which eliminates the situations 5 and 6 from the **Un** of Table 3. A proposition *P* is necessarily true in **Un** iff /P/ = **Un**; *P* is necessarily false in **Un** iff /P/ = Ø. Thus, if *A* is necessarily true, the valuations 2, 4, 6 and 8 are inadmissible and are removed from **Un**. If *A* is necessarily false, the valuations 1, 3, 5 and 7 are inadmissible and thus removed.

Un ≠ Ø iff **L** contains at least one proposition *P*. If **L** contains precisely one proposition *P* and *P* is contingent, there are two admissible situations, one in which *P* is true and one in which *P* is false, but if *P* is either necessarily true or necessarily false, there is only one

[1] Both the notion and the term *Valuation Space* were introduced in Van Fraassen (1971), where, however, the notion was left underdeveloped.

admissible situation in the corresponding **Un**. **Un** = Ø iff **L** = Ø (**L** contains no proposition). No logical relation holds when **Un** = Ø.

4. Constructing the Square from SMPL

Table 4 lists the logical relations taken into account in the present study (plus logical independence) defined for arbitrary propositions *P* and *Q*. Note that SMPL contains only the relations of equivalence and contradictoriness. The remaining relations become relevant in the context of the construction of the Square and the Logical Hexagon. To enhance formal precision, the relations of entailment, contrariety and subcontrariety have each been given a *strict* counterpart, imposing further conditions and corresponding more closely to the intuitive concepts. The 'strict' relations all 'meta-entail' their namesakes without the qualification 'strict', since the former are subject to an extra condition. The relations of entailment, contrariety and sub-contrariety as they hold in the logical systems discussed here are all of the 'strict' kind, provided the proposition types involved are neither necessarily true nor necessarily false (see below).

Table 4

Relation	**Symbol**	**Definition**
entailment	$P \vdash Q$	$/P/ \subseteq /Q/$
strict entailment	$P \mid\!\vdash Q$	$/P/ \subset /Q/$
equivalence	$P \equiv Q$	$/P/ = /Q/$
contrariety	$P >< Q$	$/P/ \cap /Q/ = \emptyset$
strict contrariety	$P \mid\!>< Q$	$/P/ \cap /Q/ = \emptyset$; $/P/ \cup /Q/ \neq \mathbf{Un}$
subcontrariety	$P \underline{><} Q$	$/P/ \cup /Q/ = \mathbf{Un}$
strict subcontrariety	$P \mid\!\underline{><} Q$	$/P/ \cup /Q/ = \mathbf{Un}$; $/P/ \cap /Q/ \neq \emptyset$
contradictoriness	$P >\!\mid\!< Q$	$/P/ \cap /Q/ = \emptyset$; $/P/ \cup /Q/ = \mathbf{Un}$
(logical independence	no symbol	$/P/ \cap /Q/ \neq \emptyset \neq /P/ \neq /Q/$; $/P/ \cup /Q/ \neq \mathbf{Un}$)

The construction of the classic Square from SMPL results from leaving out of consideration valuation 4 of Table 1, thus reducing **Un** to {1,2,3}. It is sometimes said that all that is needed to make the Square logically sound is replacing the standard definition of $\forall$ as given in(1a) with (2), which adds the condition that $\llbracket R \rrbracket \neq \emptyset$:

(2) $\llbracket \forall \rrbracket = \{<\llbracket R \rrbracket, \llbracket M \rrbracket> \mid \llbracket R \rrbracket \subseteq \llbracket M \rrbracket \text{ and } \llbracket R \rrbracket \neq \emptyset\}$

Yet, although this removes the defect of UEI, it also produces a logic that is different from both SMPL and the Square. The added condition that ⟦R⟧ ≠ Ø makes **A** false when ⟦R⟧ = Ø, so that space 4 is not left blank but reserved for **¬A**, **¬A***, **¬I** and **¬I***. This gives rise to a fully consistent alternative predicate logic, which I have dubbed *Aristotelian-Abelardian predicate logic* (AAPL), as it reflects Aristotle's original system, rediscovered and reconstructed by the medieval French philosopher Peter Abelard (1079–1142) (see Seuren 2010, Ch. 5; 2013, Section 8.4.1). In AAPL (see Figure 2), the Conversions do not hold but are replaced with the one-way (strict) entailments **A** ||– **¬I*** and **I** ||– **¬A***. Moreover, subcontrariety disappears: in AAPL: **I** and **I*** are no longer subcontraries but are logically independent, because, as pointedly observed by Abelard, if ⟦R⟧ = Ø, both **I** and **I*** are false. The construction of the classic Square from SMPL is thus brought about not by adding the condition that ⟦R⟧ ≠ Ø to the semantics of ∀ but by restricting **Un** to those situations where ⟦R⟧ ≠ Ø.

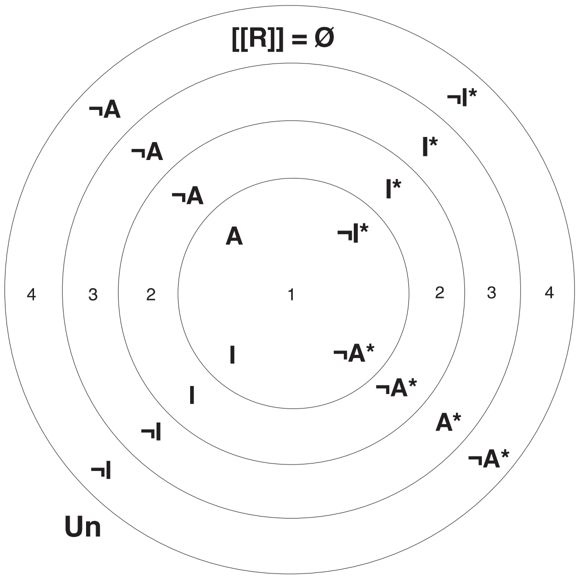

Figure 2 The VS-model for AAPL

When the situations where ⟦R⟧ = Ø are removed from **Un** and neither **A** nor **I** is either necessarily true or necessarily false, the Conversions are unaffected and, in addition:

A $|\!\vdash$ **I**, since /**A**/ = {1} and /**I**/ = {1,2} and {1} $\subset$ {1,2};

A $|\!><$ **¬I**, since {/**A**/ = {1} and /**¬I**/ = {3} and 1} $\cap$ {3} = Ø and {1} $\cup$ {3} ≠ **Un**;

¬I $|\!\vdash$ **¬A**, since /**¬I**/ = {3}, and /**¬A**/ = {2,3} and {3} $\subset$ {2,3};

I $\underline{|\!><}$ **I***, since /**I**/ = {1,2} and /**I***/ = {2,3} and {1,2} $\cup$ {2,3} = **Un** and {1,2} $\cap$ {2,3} ≠ Ø.

The picture is modified, though in principle kept intact, when one or more of the proposition types involved is either necessarily true or necessarily false. For example, if **A** is necessarily true, the spaces 2 and 3 are removed from the **Un** of both SMPL and the Square, leaving only the spaces 1 and 4 for SMPL and only space 1 for the Square. This has the effect of strengthening some of the logical relations involved or of creating new logical relations. In SMPL, for example, although contingent **A** and **I** are logically independent, **I** $|\!\vdash$ **A** when **A** is necessarily true (as in *All boys are male*), since, in that case, **Un** = {1,4}, /**I**/ = {1} and /**A**/ = {1,4}. Or, in the Square, when **A** is necessarily true, so that **Un** is restricted to just {1}, **A** $\equiv$ **I** since /**A**/ = {1} and /**I**/ = {1}, and **A** $>\!|\!<$ **¬I** since /**A**/ = {1} and /**¬I**/ = Ø, and likewise **I** $>\!|\!<$ **I*** since /**I**/ = {1} and /**I***/ = Ø. The subaltern entailments in the Square from **A** to **I** and from **¬I** to **¬A** (= **I***) are thus strengthened to equivalences, and both contrariety and subcontrariety are strengthened to contradictoriness. This leaves the Square intact for the non-strict versions of the relations involved. In what follows, however, I will leave extreme cases where a proposition type is either necessarily true or necessarily false out of consideration and limit the discussion to contingent **A** and **I**, as extreme cases, with their specific logical properties, fall outside LNL (Seuren 2010, Ch. 3).

The logical relations of the Square are standardly represented in the form of the classic Square (hence its name), as in Figure 3–b. (In polygonal representations such as Figure 3–b, '>' stands for entailment, '=' for equivalence, 'C' for contrariety, 'SC' for subcontrariety, and a cross or star in the middle for contradictoriness.) We can construct a VS-model for the Square merely by taking Figure 1 and leaving space 4 empty, as in Figure 3–a (or by leaving out space 4, as in Figure 6–a). To facilitate checking the logical relations at issue, each proposition type in Figure 3–b has its VS specified for it.

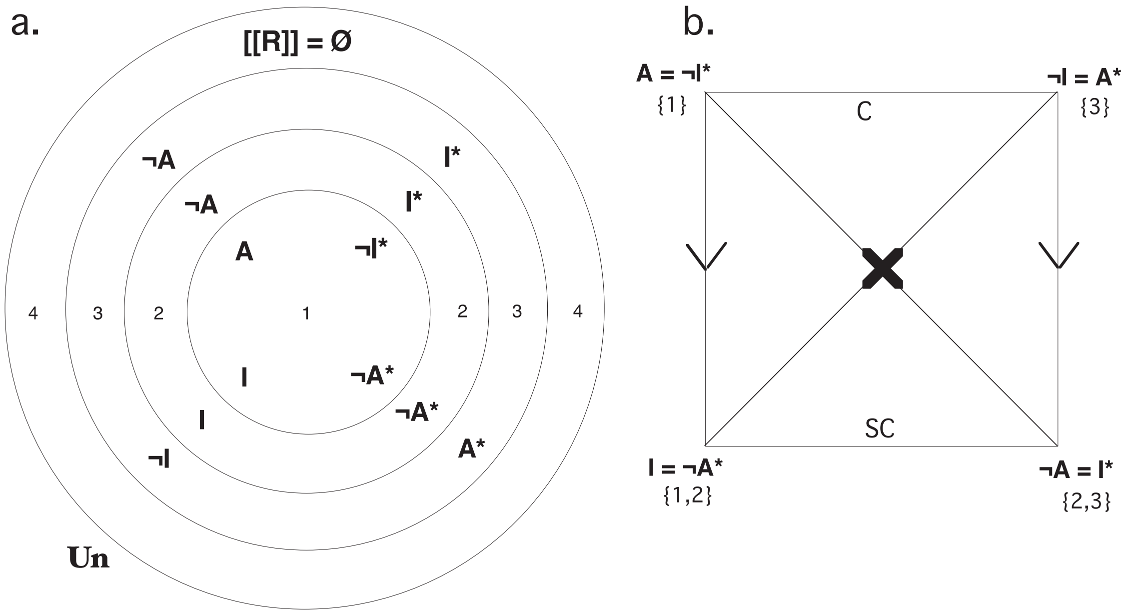

Figure 3 (a) VS-model and (b) square representation of the Square

5. Adding the Y-type

The 19th-century Scottish philosopher Sir William Hamilton (1788–1856) proposed (Hamilton 1866) to interpret the natural language word *some* as 'some but not all', rather than as the standard 'some perhaps all' of SMPL and the Square. [2] In this he was followed by the Danish linguist Otto Jespersen (1860–1943) in Jespersen (1917, 1924). I call this more restricted form of existential quantification *strict existential quantification* and, following a tradition established by the French philosopher Robert Blanché (1898–1975), I use the type name **Y** for the corresponding proposition type. The strict existential quantifier, which I write as $|\exists$, is defined as follows:

(3) $[\![|\exists]\!] = \{ \langle [\![R]\!], [\![M]\!] \rangle \mid [\![R]\!] \cap [\![M]\!] \neq \emptyset, [\![R]\!] \not\subseteq [\![M]\!] \}$

[2] Hamilton's logic is characterized further by his theory of so-called 'quantification of the predicate', in virtue of which a sentence like *All men are mortal* should be analyzed as 'all men are some mortal'. This aspect of Hamilton's logic is left out of account here, as it reflects a notion of logical structure that is totally at odds with the approach followed here and does not seem to contribute to a better insight into the logical aspects of language or cognition.

Hamilton's proposal, which was rejected and even treated with derision by mainstream logicians, amounts to a triadic predicate logic consisting of the three mutually contrary proposition types **A**, **¬I** and **Y**, forming what is known as the *Hamiltonian Triangle of Contraries*.

Contrary to the now widespread assumption that the interpretation 'some but not all' is a pragmatic derivate from the standard 'some perhaps all', I submit, following Hamilton (1866), Ginzberg (1913), Jespersen (1917, 1924), Jacoby (1950, 1960), Sesmat (1951), Blanché (1953, 1966), that $|\exists$ deserves a place in predicate logic on a par with $\forall$ and $\exists$. In fact, the addition of the **Y**-type (plus its negative compositions) and the corresponding quantifier $|\exists$ makes for a nontrivial enrichment of both SMPL and the Square, while opening up new possibilities for the study of the natural logic of language and cognition (see Seuren & Jaspers, forthcoming; Seuren, forthcoming). The addition to Table 1 of the **Y**-type plus its three negative compositions as specified in Table 5 thus increases the number of proposition types from 8 to 12. (NB: **U** is often, following Blanché 1953, 1966, used for **¬Y** ≡ **¬Y***).

Table 5

type name	**linguistic expression**	**formal language**
Y	Some but not all R is M	$|\exists$(Rx,Mx)
¬Y (= **U**)	All R is M *OR* No R is M	¬[$|\exists$(Rx,Mx)]
Y*	Some but not all R is not M	$|\exists$(Rx,¬Mx)
¬Y* (= **U**)	All R is M *OR* No R is M	¬[$|\exists$(Rx,¬Mx)]

Given the satisfaction condition for $|\exists$ as specified in (3) above, it follows that **Y** is valued T in valuation 2 and F in the valuations 1, 3 and 4 of Table 2 above. Moreover, it follows that **Y** ≡ **Y***, since if $[\![R]\!] \cap [\![M]\!] \neq \emptyset$ and $[\![R]\!] \not\subseteq [\![M]\!]$, then $[\![R]\!] \cap [\![\overline{M}]\!] \neq \emptyset$ and $[\![R]\!] \not\subseteq [\![\overline{M}]\!]$, and vice versa. Consequently, **¬Y** ≡ **¬Y***.

The extension of SMPL with **Y** and its negative compositions results in the VS-model of SMPL in Figure 4. Figure 5 shows (a) the dismantled square of standard SMPL, consisting of only the Conversions and the contradictories of the terms concerned, and (b) the incomplete hexagon resulting from the addition of **Y** and its negative compositions, as it can be read off Figure 4.

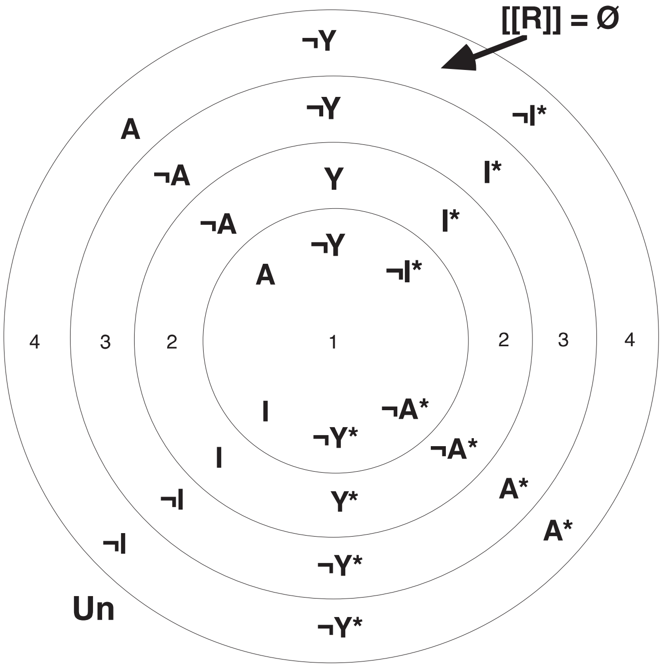

*Figure 4 The VS-model for SMPL extended with **Y** and its negative compositions*

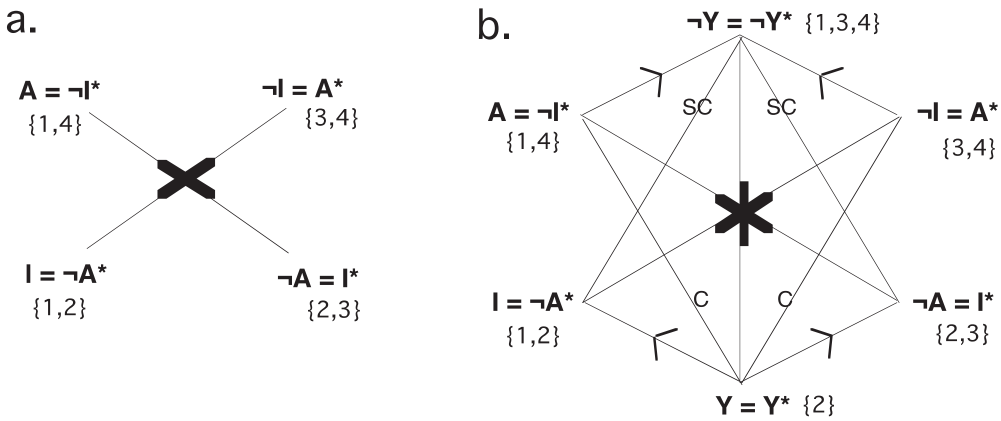

*Figure 5 The dismantled square for SMPL and the incomplete hexagon resulting from the incorporation of **Y** and its negative compositions*

Figure 5 shows how the incorporation of **Y** and its negative compositions leads to a considerable enrichment of SMPL. An even greater enrichment results if the Square is the beneficiary of the incorporation of **Y** and its negative compositions—that is, when space 4 is disregarded. Whereas SMPL in its original form consists of only the Conversions, and the addition of **Y** and its negative compositions to SMPL greatly increases the number of instantiated logical relations, as shown in Figure 5–b, the incorporation of **Y** and its negative compositions into the Square makes the hexagon complete, yielding the Logical Hexagon as envisaged by Jacoby, Sesmat and Blanché (Figure 6–b). Disregarding cases where ⟦R⟧ = Ø is thus seen to lead systematically to what is probably (failing a theory of 'logical power') a maximal increase in the logical power of the system at hand.

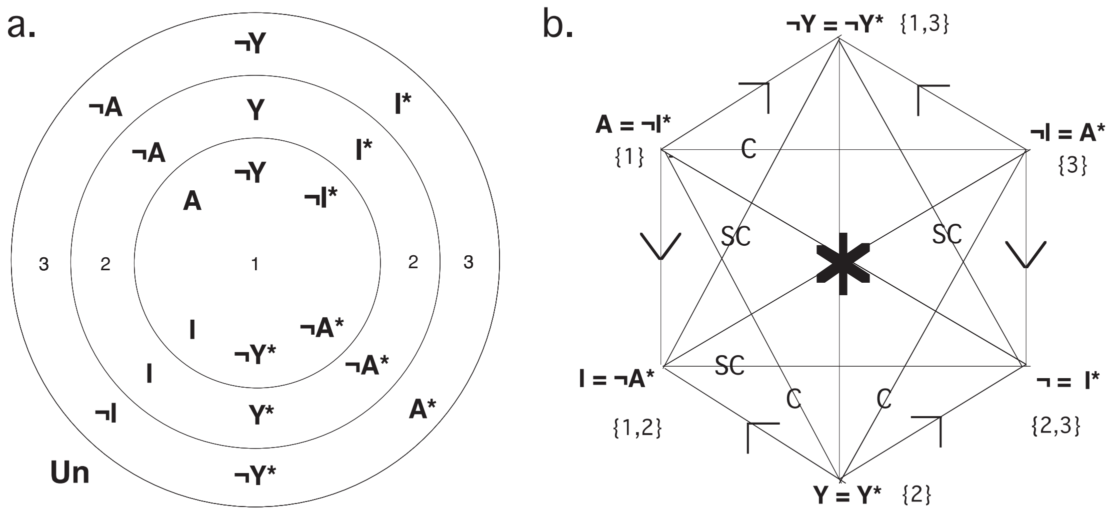

Figure 6 VS-model and corresponding polygonal representation for the Hexagon

One notes that the Logical Hexagon contains the Hamiltonian *Triangle of Contraries* <**A**,**Y**,**¬I**>, the *Triangle of Subcontraries* <**I**,**¬A**,**¬Y**>, the classic Aristotelian-Boethian *Square of Opposition* <**A**,**I**,**¬I**,**¬A**> and thus SMPL to the extent that it is valid for the spaces 1,2 and 3. The addition of the **Y**-type plus its negative compositions to these logical systems is thus seen to make good logical sense.[3]

[3] This was already seen by S. Ginzberg (1913), whose arguments were sharper than Hamilton's. (It has so far proved impossible to trace this author, unless he is identical—which is doubtful—with the Salomon or Simon Ginzberg who acted as Albert Einstein's *ad hoc* secretary during the latter's trip through the United States in 1921.) Ginzberg's little article provoked an acerbic reaction by Louis Couturat (1868–1914), a well-known follower of Russell, who wrote (Couturat 1913: 256; translation mine): "The problem raised by Mr Ginzberg […] was solved a long time ago by formal logicians in a way that seems to me to be satisfactory and decisive." Ginzberg retorted (Ginzberg 1914) that no logical principle prohibits the introduction of a quantifier meaning 'some but not all' and that such a quantifier, being more precise than 'some perhaps all', has certain functional advantages. In his short final repartee (Couturat 1914), Couturat had nothing better to say than (translation mine) "[O]ne steps out of the framework of classical logic the moment one tries to make propositions 'quantitatively' precise. One may well elaborate alternative legitimate

The **Y**-type plus its negative compositions may also be incorporated into AAPL. Doing so leads to the VS-model shown in Figure 7. A polygonal representation corresponding to the VS-model may also be set up, but, due to the scarcity of equivalences, this proves unrewarding.

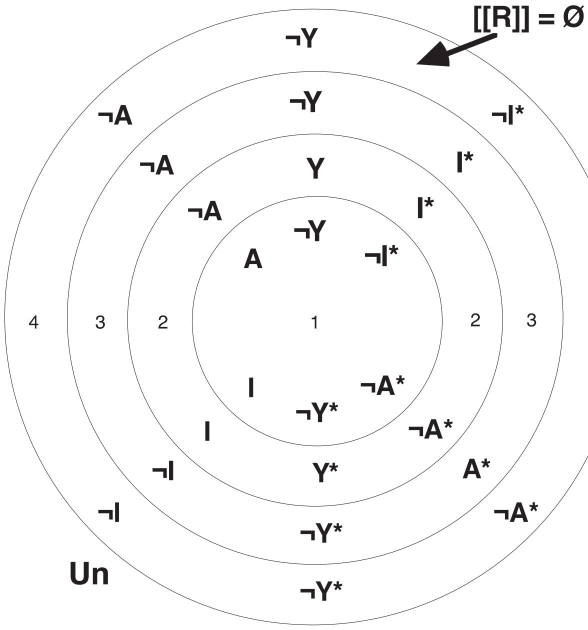

*Figure 7 The VS-model for AAPL extended with **Y** and its negative compositions*

6. Conclusion and further perspectives

The question of what logical system is best taken to correspond to natural human logic is still far removed from a final answer. But a few things are clear. First, the question is of an empirical nature. One empirical touchstone is formed by intuitive judgements of necessary consequence and of consistency through texts, just as intuitive judgements regarding given categories of data are an empirical touchstone for many other human sciences, such as linguistics. Other such empirical touchstones may be found in

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and consistent systems, but one cannot make the classical system more 'perfect' by introducing greater 'precision', which is alien to its spirit." It should be clear that this final reply by Couturat was gratuitous and that, far from being alien to it, it is entirely in the spirit of the "classical system" to make it more perfect by introducing greater precision.

psychological experiments, but, to my knowledge, none have so far been carried out or devised.

Then, it seems that **Un** restriction must be taken to be a central feature of LNL. In the present study, static **Un** restriction has been the central topic, but the results presented here, interesting and intriguing as they may be, are far from complete: there are no doubt further aspects to this form of **Un** restriction that need elaboration and elucidation. The dynamic (presuppositional) and developmental (cognitive) forms of **Un** restriction also need to be investigated at greater depth and confirmed, if possible, by experimental results. And, importantly, a clearer picture is needed of how these three forms of **Un** restriction relate to each other and collaborate to secure consistency in texts.

This, it seems to me, is one fruitful way of going about a long overdue investigation of the relation between logic on the one hand and the human cognitive-linguistic complex on the other. Another fruitful way is the precise investigation of phenomena of logical scope, which have been given a very poor treatment in current linguistic theories even though their repercussions on the study of language in a general sense are momentous and all-pervasive. The neglect of logic that has characterized the study of language and of the mind over the past half century has done these disciplines no good.

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